CHAPTER 2

Polynomials

1. OBJECTIVE QUESTIONS

1. The value of x, for which the polynomials x^2-1 and x^2-2x+1 vanish simultaneously, is

(a) 2 (b) -2

(c) -1 (d) 1

Ans : (d) 1

The expressions $(x-1)(x+1)$ and $(x-1)(x-1)$ which vanish if $x=1$

2. If α and β are zeroes and the quadratic polynomial $f(x)=x^2-x-4$, then the value of $\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta$ is

(a) $\frac{15}{4}$ (b) $\frac{-15}{4}$

(c) 4 (d) 15

Ans : (a) $\frac{15}{4}$

Given that, $f(x)=x^2-x-4$

$$\alpha+\beta=1 \text{ and } \alpha\beta=-4$$

We have, $\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta=\frac{\alpha+\beta}{\alpha\beta}-\alpha\beta=-\frac{1}{4}+4=\frac{15}{4}$

3. The value of the polynomial $x^8-x^5+x^2-x+1$ is

(a) positive for all the real numbers

(b) negative for all the real numbers

(c) 0

(d) depends on value of x

Ans : (a) positive for all the real numbers

Let $f(x)=x^8-x^5+x^2-x+1$

For $x=1$ or 0

$f(x)=1>0$

For $x<0$

each term of $f(x)$ is Positive and so first $f(x)>0$. Hence, $f(x)$ is Positive for all real x.

4. On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x-2$ and $-2x+4$ respectively, then $g(x)$ is equal to

(a) x^2+x+1 (b) x^2+1

(c) x^2-x+1 (d) x^2-1

Ans : (c) x^2-x+1

Here, Dividend $=x^3-3x^2+x+2$

Quotient $=x-2$

Remainder $=-2x+4$ and

Divisor $=g(x)$

Since,

dividend = Divisor × Quotient +Remainder

So, $x^3-3x^2+x+2=g(x)\times(x-2)+(-2x+4)$

$$g(x)\times(x-2)=x^3-3x^2+x+2+2x-4$$

$$g(x)=\frac{x^3-3x^2+3x-2}{x-2}$$

$$=\frac{(x-2)(x^2-x+1)}{(x-2)}$$

$$=x^2-x+1$$

5. If $x=0.\overline{7}$, then $2x$ is

(a) $1.\overline{4}$ (b) $1.\overline{5}$

(c) $1.\overline{54}$ (d) $1.\overline{45}$

Ans : (b) $1.\overline{5}$

$10x=7.\overline{7}$

or $x=0.\overline{7}$

Subtracting, $9x=7$

$$x=\frac{7}{9}$$

$$2x=\frac{14}{9}=1.555\,..........$$

$$=1.\overline{5}$$

6. The difference between two numbers is 642. When the greater is divided by the smaller, the quotient is 8 and the remainder is 19, then find the sum of cube of numbers.

(a) 391322860 (b) 319322860

(c) 319322680 (d) 391223860

Ans : (a) 391322860

Let one number be x. Then, another number be $642+x$.

Difference between two numbers = 642

By division algorithm,

Dividen = Divisor × Quotient + Remainder

Here, Dividend $=642+x$, divisor $=x$,

quotient $=8$

and remainder $=19$

$$642+x=8x+19$$

$$x-8x=19-642$$

$$-7x=-623$$

$$x=\frac{-623}{-7}=89$$

Then, other number $=642+x=642+89=731$

Hence, the required numbers are 89 and 731.

$$89^3+(731)^3=704969+390617891$$

$$= 391322860$$

7. Lowest value of $x^2 + 4x + 2$ is
(a) 0 (b) 2
(c) 2 (d) 4

Ans : (b) 2

$$x^2 + 4x + 2 = (x^2 + 4x + 2) - 2$$
$$\text{Lowest value} = -2$$
When, $x + 2 = 0$

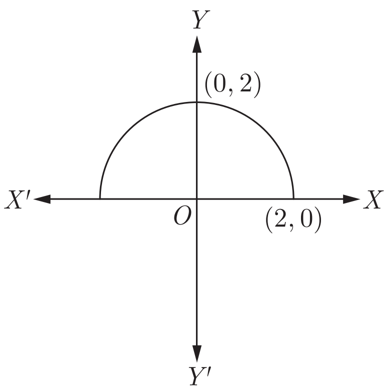
8. If $a^3 - 3a^2b + 3ab^2 - b^3$ is divided by $(a-b)$, then the remainder is
(a) $a^2 - ab + b^2$ (b) $a^2 + ab + b^2$
(c) 1 (d) 0

Ans : (d) 0

9. A quadratic polynomial when divided by x+2 leaves a remainder of 1 and when divided by x-1, leaves a remainder of 4. What will be the remainder if it is divided by $(x+2)(x-1)$?
(a) 1 (b) 4
(c) $x + 3$ (d) $x - 3$

Ans : (c) $x + 3$

10. If a quadratic polynomial curve in the shape of semi-circle is shown below.



Then, the equation of this curve.
(a) $-x^2 + 2$ (b) $x^2 + 2$
(c) $\frac{1}{2}x^2 + 2$ (d) $-\frac{1}{2}x^2 + 2$

Ans : (d) $-\frac{1}{2}x^2 + 2$

Let quadratic polynomial be,

$$f(x) = ax^2 + bx + c \qquad ...(1)$$

The coordinates of given curve are $(-2, 0), (2, 0)$ and (0,2)

On putting the coordinates in Eq. (1), we get

$$0 = a(-2)^2 + b(-2) + c$$
$$0 = 4a - 2b + c \qquad ...(2)$$
$$0\ a(2)^2 + b(2) + c$$
$$0 = 4a + 2b + c \qquad ...(3)$$
and $$2 = a(0)^2 + b(0) + c \Rightarrow c = 2 ...(4)$$

On solving Eq. (2), (3) and (4), we get

$$a = -\frac{1}{2},\ b = 0 \text{ and } c = 2$$

On putting the values of a, b and c in Eq. (1), we get

$$f(x) = -\frac{1}{2}x^2 + 0x + 2$$
$$f(x) = -\frac{1}{2}x^2 + 2$$

11. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is
(a) 2 (b) -2
(c) 4 (d) -4

Ans : (c) 4

$$\text{Sum of the zeroes} = \frac{3k}{2}$$
$$6 = \frac{3k}{2}$$
$$k = \frac{12}{3} = 4$$

12. If a cubic polynomial with the sum of its zeroes, sum of the products and its zeroes taken two at a time and product of its zeroes as 2, -5 and -11 respectively, then the cubic polynomial is
(a) $x^3 + 7x - 6$ (b) $x^3 + 7x + 6$
(c) $x^3 - 7x - 6$ (d) $x^3 - 7x + 6$

Ans : (d) $x^3 - 7x + 6$

Let α, β, γ be the zeros of the required polynomials

$$\alpha + \beta + \gamma = 0$$
$$\alpha\beta + \beta\gamma + \alpha\gamma = -7$$
$$\alpha\beta\gamma = -6$$

Required cubic polynomial is

$k\left[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\right]$

where k is non-zero constant

$k\left[x^3 + (0)x^2 + (-7)x - (-6)\right] = x^3 - 7x + 6$

[consider, $k = 1$]

13. If α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then the value of $\alpha^4 + \beta^4$ is

(a) $\frac{(b^2 - 2ac)^2 + a^2c^2}{a^4}$ (b) $\frac{(b^2 + 2ac)^2 - a^2c^2}{a^4}$

(c) $\frac{(b^2 - 2ac)^2 - 2a^2c^2}{a^4}$ (d) $\frac{(b^2 + 2ac)^2 + 2a^2c^2}{a^4}$

Ans : (c) $\frac{(b^2 - 2ac)^2 - 2a^2c^2}{a^4}$

Given, α and β are the zeroes of the polynomial

$$f(x) = ax^2 + bx + c$$

Sum of zeroes, $\alpha + \beta = \frac{-b}{a}$

and produce of zeroes,

$$\alpha\beta = \frac{c}{a}$$

Now, $$\alpha^4 + \beta^4 = (\alpha^2)^2 + (\beta^2)^2$$
$$= (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$$
$$[a^2 + b^2 = (a+b)^2 - 2ab]$$
$$= \left[(\alpha + \beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2$$

On substituting, $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ in the above equation, we get

$$\alpha^4+\beta^4 = \left[\left(\frac{-b}{a}\right)^2-2\left(\frac{c}{a}\right)\right]^2-2\left(\frac{c}{a}\right)^2$$
$$= \left[\frac{b^2}{a^2}-\frac{2c}{a}\right]^2-\frac{2c^2}{a^2}$$
$$= \left[\frac{b^2-2ac}{a^2}\right]^2-\frac{2c^2}{a^2}$$
$$= \frac{(b^2-2ac)^2}{a^4}-\frac{2c^2}{a^2}$$
$$= \frac{(b^2-2ac)^2-2a^2c^2}{a^4}$$

14. The polynomial $f(x)=ax^3+bx-c$ is divisible by the polynomial $g(x)=x^2+bx+c$, $c\neq 0$, if

(a) $ab=2$ (b) $ab=1$

(c) $ac=2$ (d) $c=2b$

Ans : (b) $ab=1$

If ax^3+bx-c is exactly divisible by x^2+bx+c, then the remainder should be zero.

On dividing, we get

$$
\begin{array}{rl}
 & ax-ab \\
x^2+bx+c \,\big) & ax^3 \qquad +bx-c \\
 & \underline{\overset{-}{ax^3}\ \overset{-}{+abx^2}\ \overset{-}{+acx}} \\
 & -abx^2+(b-ac)x-c \\
 & -abx^2-ab^2x-abc \\
 & \underline{\quad + \qquad + \qquad + \quad} \\
 & (ab^2+b-ac)x+abc-c
\end{array}
$$

$$\text{Quotient} = ax-ab \text{ and,}$$
$$\text{remainder} = (ab^2+b-ac)x+abc-c = 0$$
$$(ab^2+b-ac)x+abc-c = 0$$

Comparing the coefficient of x and constant term both sides, we get

$$ab^2+b-ac = 0$$
$$\text{and} \qquad abc-c = 0$$
$$ab = 1$$

15. If one of the zeroes of a quadratic polynomial of the form x^2+ax+b is the negative of the other, then which of the following is correct?

(a) Polynomial has linear factors

(b) Constant term of polynomial is negative

(c) Both (a) and (b) are correct

(d) Neither (a) nor (b) is correct

Ans : (c) Both (a) and (b) are correct

Let given polynomial be, $p(x)=x^2+ax+b$

Again, let α and β be the zeroes of $p(x)$.

Then,

$$\text{product of zeroes} = \frac{\text{Constanterm}}{\text{Coefficient of } x^2}$$
$$\alpha\beta = \frac{b}{1} \Rightarrow \alpha\beta=b \qquad \text{...(1)}$$

Since, one of the zeroes of the quadratic polynomial $p(x)$ is negative of the other.

$$\alpha\beta < 0 \Rightarrow b<0 \qquad \text{[from Eq. (1)]}$$

So, b should be negative.

As, $(x-\alpha)$ and $(x-\beta)$ are the factors of polynomial $p(x)$.

Then,
$$p(x) = (x-\alpha)(x-\beta)$$
$$= (x-\alpha)(x+\alpha) \qquad [\beta=-\alpha]$$
$$= x^2-\alpha^2 = x^2-k$$
$$[\text{where, } \alpha^2=k \text{ is positive term}]$$

So, $\qquad x^2+ax+b = x^2-k$

Then, $a=0$ and $(x+\sqrt{k})$, $(x-\sqrt{k})$ are the linear factors of polynomial $p(x)$.

Hence, if one of the zeroes of quadratic olynomial $p(x)$ is the negative of the other, then it has linear factor and the constant term is negative, i.e. $b<0$.

16. If α,β and γ are the zeroes of the polynomial $p(x)=ax^3+3bx^2+3cx+d$ and having relation $2\beta=\alpha+\gamma$, then $2b^3-3abc+a^2d$ is

(a) -1 (b) 1

(c) 0 (d) None of the above

Ans : (c) 0

Given, $\qquad p(x) = ax^3+3bx^2+3cx+d$

On comparing with Ax^3+Bx^2+Cx+D, we get

$$A = a,\ B=3b,\ C=3c \text{ and } D=d$$

Then, sum of zeroes,

$$\alpha+\beta+\gamma = -\frac{B}{A} = -\frac{3b}{a} \qquad \text{...(1)}$$

Product of zeroes taken two at a time,

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{C}{A}$$
$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{3c}{a} \qquad \text{...(2)}$$

and product of all zeroes,

$$\alpha\beta\gamma = \frac{-D}{A} \Rightarrow \alpha\beta\gamma=-\frac{d}{d} \qquad \text{...(3)}$$

Also,
$$2\beta = \alpha+\gamma \qquad \text{[given]}$$
$$2\beta = \frac{-3b}{a}-\beta \qquad \text{[From Eq. (1)]}$$
$$3\beta = -\frac{3b}{a} \Rightarrow \beta=\frac{-b}{a}$$

From eq. (3),

$$(\alpha\gamma)\beta = \frac{-d}{a}$$
$$\alpha\gamma = \frac{-d}{a\beta} = \frac{-d}{a(-b/a)} \qquad \left[\beta=\frac{-b}{a}\right]$$
$$\alpha\gamma = \frac{d}{b}$$

From Eq. (2),

$$\beta(\alpha+\gamma)+\gamma\alpha = \frac{3c}{a}$$
$$\beta\times 2\beta+\gamma\alpha = \frac{3c}{a} \qquad [\alpha+\gamma=2\beta]$$
$$2\beta^2+\frac{d}{b} = \frac{3c}{a} \qquad \left[\alpha\gamma=\frac{d}{b}\right]$$
$$2\left(\frac{-b}{a}\right)^2+\frac{d}{b} = \frac{3c}{a} \qquad \left[\beta=\frac{-b}{a}\right]$$
$$\frac{2b^2}{a^2}+\frac{d}{b} = \frac{3c}{a}$$

$$\frac{2b^3 + a^2 d}{a^2 d} = \frac{3c}{a}$$

$$2b^3 + a^2 d = \frac{3a^2 bc}{a} = 3abc$$

$$2b^3 - 3abc + a^2 d = 0 \qquad \text{Hence proved.}$$

17. If the square of difference of the zeroes of the quadratic polynomial $x^2 + px + 45$ is equal to 144, then the value of p is

(a) ± 9 (b) ± 12

(c) ± 15 (d) ± 18

Ans : (d) ± 18

Given that, $f(x) = x^2 + px + 45$

Then, $\alpha + \beta = \frac{-p}{1} = -p$

and $\alpha\beta = \frac{45}{1} = 45$

According to given condition,

$$(\alpha - \beta)^2 = 144$$

$$(\alpha + \beta)^2 - 4\alpha\beta = 144$$

$$(-p)^2 - 4(45) = 144$$

$$p^2 = 144 + 180$$

$$p^2 = 324 \Rightarrow p = \pm 18$$

18. If α and β are zeroes and the quadratic polynomial $p(S) = 3S^2 + 6S + 4$, then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$ is

(a) 7 (b) 6

(c) 8 (d) 10

Ans : (c) 8

Since, α and β are the zeroes of the polynomial $p(S) = 3S^2 - 6S + 4$.

$$\alpha + \beta = \frac{-(-6)}{3} = 2 \text{ and } \alpha\beta = \frac{4}{3}$$

We have, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + \frac{2(\alpha + \beta)}{\alpha\beta} + 3\alpha\beta$$

$$= \frac{(2)^2 - 2 \times \frac{4}{3}}{\frac{4}{3}} + \frac{2 \times 2}{\frac{4}{3}} + 3 \times \frac{4}{3}$$

$$= \frac{4 - \frac{8}{3}}{\frac{4}{3}} + 3 + 4 = \frac{4}{4} + 7$$

$$= 1 + 7 = 8$$

19. Find the zeroes of the quadratic polynomial $y^2 - 3y + 2$ with the help of the graph.

(a) $1, -2$ (b) $\frac{-1}{4}, \frac{3}{2}$

(c) $6, -1$ (d) $1, 2$

Ans : (d) 1, 2

Let the given quadratic polynomial be

$$x = y^3 - 3y + 2$$

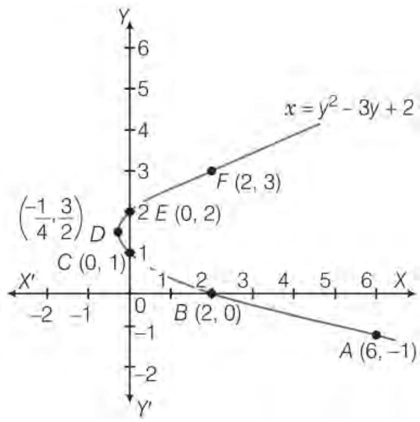
Here, we see that given polynomial is in y variable. In this case, the intersection point on Y-axis is the required zeroes of the given polynomial. To draw its graph, we need some different values of x corresponding to different value of y.

Then, we get the following table:

x	6	2	0	$-1/4$	0	2
y	-1	0	1	3/2	2	3

Now,plot all the points on the graph paper and join them. Thus, we get the graph of $y^3 - 3y + 2$.

The intersection points on Y-axis are two distinct points whose y-coordinates are (0,1) and (0,2).



It is clear from the graph that 1 and 2 are zeroes of given polynomial.

20. If the sum of the zeroes of the equation $\frac{1}{x + a} + \frac{1}{x + b} = \frac{1}{c}$ is zero, then the product of zeroes of the equation is?

(a) $\frac{a^2 + b^2}{2}$ (b) $\frac{-(a^2 + b^2)}{2}$

(c) $\frac{ab}{2}$ (d) $\frac{(a + b)^2}{2}$

Ans : (b) $\frac{-(a^2 + b^2)}{2}$

Given equation is

$$\frac{1}{x + a} + \frac{1}{x + b} = \frac{1}{c}$$

$$\frac{x + b + x + a}{(x + a)(x + b)} = \frac{1}{c}$$

$$c(2x + a + b) = (x + a)(x + b)$$

$$2cx + (a + b)c = x^2 + (a + b)x + ab$$

$$x^2 + (a + b - 2c)x + ab - ac - bc = 0$$

Let the zeroes of the above equation be α and β.

Given, $\alpha + \beta = 0 \Rightarrow \frac{(a + b - 2c)}{1} = 0$

$$a + b = 2c \qquad \text{...(1)}$$

Now, product of zeroes,

$$\alpha\beta = \frac{(ab - ac - bc)}{1}$$

$$= ab - (a + b)c$$

$$= ab - (a+b)\left(\frac{a+b}{2}\right)$$

[From Eq. (1)]

$$= \frac{2ab - (a+b)^2}{2}$$

$$= \frac{2ab - (a^2 + b^2 + 2ab)}{2}$$

$$= -\frac{(a^2 + b^2)}{2}$$

21. Draw the graph of the polynomial $-x^2 + x + 2$ and find the maximum value of the polynomial.

(a) 2 (b) $\frac{5}{2}$

(c) $\frac{9}{4}$ (d) None of these

Ans : (c) $\frac{9}{4}$

Let the given quadratic polynomial be

$$y = -x^2 + x + 2$$

On comparing with $ax^2 + bx + c$, we get

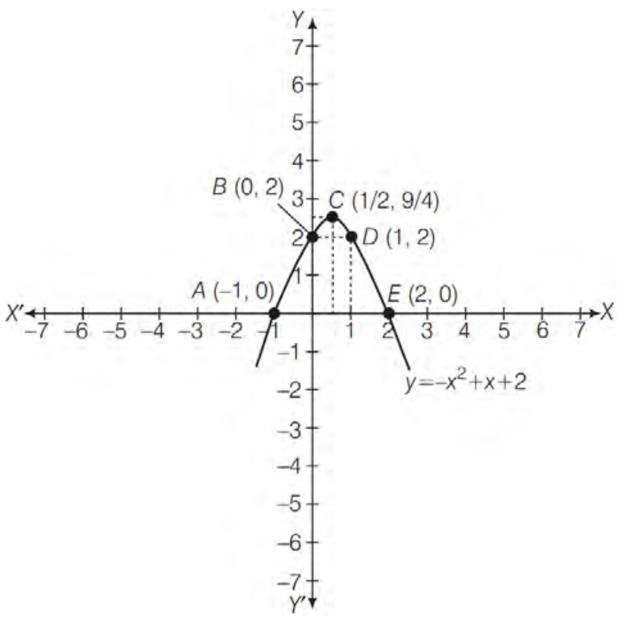
$$a = -1,\ b = 1 \text{ and } c = 2$$

Here, $a = -1 < 0$, so the shape of the parabola is opening downward. To draw its graphs, we need some different values of y corresponding to different values of x.

Then, we get the following table.

x	-1	0	1/2	1	2
y	0	2	9/4	2	0

Now, plot all the points on the graph and join them. Thus, we get the graph of $-x^2 + x + 2$.



Maximum value of the polynomial

$$= \frac{-(b^2 - 4ac)}{4a}$$

$$= \frac{-[(1)^2 - 4 \times (-1) \times 2]}{4 \times (-1)}$$

$$= \frac{-(1+8)}{-4} = \frac{9}{4}$$

2. FILL IN THE BLANK

1. A .... polynomial is of degree one.
Ans : Linear

2. A cubic polynomial is of degree..........
Ans : Three

3. We get the original number if we multiply the .... together.
Ans : Factors

4. Degree of remainder is always .... than degree of divisor.
Ans : Smaller/less

5. .... equation is valid for all values of its variables.
Ans : Identity

6. Polynomials of degrees 1, 2 and 3 are called .... , ... and .... polynomials respectively.
Ans : linear, quadratic, cubic

7. .... is not equal to zero when the divisor is not a factor of dividend.
Ans : Remainder

8. The zeroes of a polynomial $p(x)$ are precisely the $x-$ coordinates of the points, where the graph of $y = p(x)$ intersects the .... axis.
Ans : x

9. The algebraic expression in which the variable has non-negative integral exponents only is called ....
Ans : Polynomial

10. A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most .... zeroes.
Ans : 3

11. A .... is a polynomial of degree 0.
Ans : Constant

12. If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then $\alpha + \beta = \frac{-b}{..........}$ and $\alpha\beta = \frac{c}{..........}$
Ans : a, a

13. If α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d = 0$, then $\alpha + \beta + \gamma = \frac{-b}{..........}$
Ans : a

14. The highest power of a variable in a polynomial is called its ....
Ans : Degree

15. A liner polynomial is represented by a ....
Ans : Straight line

16. Zero of a polynomial is always ....
Ans : zero

17. A polynomial of degree n has at the most .... zeroes.
Ans : n

3. TRUE/FALSE

1. A polynomial of degree n has exactly n zeros.
Ans : True

2. 3, -1, 1/3 are the zeroes of the cubic polynomial
$$p(x) = 3x^3 - 5x^2 - 11x - 3$$
Ans : True

3. Number of zeros that polynomial $f(x) = (x-2)^2 + 4$ can have is three.
Ans : False

4. A cubic polynomial has atleast one zero.
Ans : False

5. $\frac{1}{\sqrt{5}}x^{\frac{1}{2}} + 1$ is a polynomial
Ans : False, because the exponent of the variable is not a whole number.

6. $(z-1)$ is a factor of $g(z) = 2z^3 - 2$.
Ans : True

7. Degree of a zero polynomial is not defined.
Ans : True

8. For polynomials $p(x)$ and any non-zero polynomial $g(x)$, there are polynomials q(x) and and r(x) such that
$$p(x) = g(x)q(x) + r(x),$$
where $r(x) = 0$
or degree $r(x) <$ degree $g(x)$.
Ans : True

9. A polynomial having two variables is called a quadratic polynomial.
Ans : False

10. Sum of zeroes of quadratic polynomial
$$= -\frac{(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$
Ans : True

11. A polynomial cannot have more than one zero
Ans : False, a polynomial can have any number of zeroes. It depends upon the degree of the polynomial.

12. Graph of a quadratic polynomial is an ellipse.
Ans : False

13. The degree of the sum of two polynomials each of degree 5 is always 5.
Ans : False, $x^5 + 1$ and $-x^5 + 2x + 3$ are two polynomials of degree 5 but the degree of the sum of the two polynomials is 1.

14. $\frac{6\sqrt{x} + x^{3/2}}{\sqrt{x}}$ is a polynomial, $x \neq 0$.
Ans : True, because $\frac{6\sqrt{x} + x^{3/2}}{\sqrt{x}} = 6 + x$, which is a polynomial.

15. Every polynomial equation has at least one real root.
Ans : False

16. Product of zeroes of quadratic polynomial $= -\frac{\text{constant term}}{(\text{coefficient of } x^2)}$
Ans : False

17. If $p(x) = ax + b$ then zero of $p(x)$ is $\frac{-b}{a}$.
Ans : True

18. Zeroes of quadratic polynomial $x^2 + 7x + 10$ are 2 and -5
Ans : False

19. Sum of zeroes of $2x^2 - 8x + 6$ is -4
Ans : False

20. Degree of a quadratic polynomial is less than or equal to two.
Ans : False

4. MATCHING QUESTIONS

DIRECTION : Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column-I have to be matched with statements (p, q, r, s) in column-II.

1.

	Column-I (Zeroes)		Column-II (Quadratic polynomial)
(A)	3 and -5	(p)	$x^2 - 25$
(B)	$5 + \sqrt{2}$ and $5 - \sqrt{2}$	(q)	$x^2 + 2x - 15$
(C)	-9 and $1/9$	(r)	$x^2 + (80/9)x - 1$
(D)	5 and -5	(s)	$x^2 - 10x + 21$

Ans : (A) $-$ q (B) $-$ s, (C) $-$ r, (D) $-$ p.

2.

	Column-I (Polynomial)		Column-II (Remainder)
(A)	$\frac{x^3-3x^2+x+2}{x^2-x+1}$	(p)	8
(B)	$\frac{x^3-3x^2+5x-3}{x+2}$	(q)	$x-5$
(C)	$\frac{x^4-6x^3+16x^2-25x+10}{x^2-2x+5}$	(r)	-33
(D)	$\frac{x^4-3x^2+4x+5}{x^2-x+1}$	(s)	$-2x+4$

Ans : (A) – s, (B) – r, (C) – q, (D) – p.

DIRECTION : Following questions has four statements (A, B, C and D) given in Column I and statements (p, q, r, s.....) in Column II. Any given statement in Column I can have corrent matching with one or more statements (s) given in Column II.

	Column-I (Polynomials)		Column-II (Zeroes)
(A)	$4-x^2$	(p)	7
(B)	x^3-2x^2	(q)	-2
(C)	$6x^2-3-7x$	(r)	2
(D)	$-x+7$	(s)	3/2
		(t)	0
		(u)	$-1/3$

Ans : (A) – (r, q), (B) – (r, t), (C)– (s, u), (D)– p.

1.
$$4-x^2 = 0$$
$$x = \pm 2$$

2.
$$x^3-2x^2 = 0$$
$$x^2(x-2) = 0$$
$$x = 0$$
or
$$x = 2$$

3.
$$6x^2-7x-3 = 0$$
$$6x^2-9x+2x-3 = 0$$
$$3x(2x-3)+1(2x-3) = 0$$
$$(3x+1)(2x-3) = 0$$
$$x = 3/2$$
or
$$x = -1/3$$

4.
$$x = 7$$

5. ASSERTION AND REASON

DIRECTION : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

1. **Assertion :** If α, β, γ are the zeroes of x^3-2x^2+qx-r and $\alpha+\beta=0$, then $2q=r$.
Reason : If α, β, γ are the zeroes of ax^3+bx^2+cx+d,

then
$$\alpha+\beta+\gamma = -\frac{b}{a}$$
$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a}$$
$$\alpha\beta\gamma = -\frac{d}{a}.$$

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Clearly, Reason is true. [Standard Result]
$$\alpha+\beta+\gamma = -(-2) = 2$$
$$0+\gamma = 2$$
$$\gamma = 2$$
$$\alpha\beta\gamma = -(-r) = r$$
$$\alpha\beta(2) = r$$
$$\alpha\beta = \frac{r}{2}$$
$$\alpha\beta+\beta\gamma+\gamma\alpha = q$$
$$\frac{r}{2}+r(\alpha+\beta) = q$$
$$\frac{g}{2}+\gamma(0) = q$$
$$\gamma = 2q$$ Assertion is true.

Since, Reason gives Assertion.

2. **Assertion :** $(2-\sqrt{3})$ is one zero of the quadratic polynomial then other zero will be $(2+\sqrt{3})$.
Reason : Irrational zeros (roots) always occurs in pairs.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

As irrational roots/zeros always occurs in pairs therefore, when one zero is $(2-\sqrt{3})$ then other will be $2+\sqrt{3}$. So, both A and R are correct and R explains A.

3. **Assertion :** Zeroes of $f(x)=x^2-4x-5$ are 5, -1
Reason : The polynomial whose zeroes are $2+\sqrt{3}, 2-\sqrt{3}$ is x^2-4x+7.

Ans : (c) Assertion (A) is true but reason (R) is false.

4. **Assertion :** x^2+4x+5 has two zeroes.
Reason : A quadratic polynomial can have at the most two zeroes.

Ans : (d) Assertion (A) is false but reason (R) is true.

5. **Assertion :** If one zero of poly-nominal $p(x)=(k^2+4)x^2+13x+4k$ is reciprocal of other, then $k=2$.
Reason : If $(x-\alpha)$ is a factor of $p(x)$, then $p(\alpha)=0$

i.e. α is a zero of $p(x)$.

Ans : (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Reason is true.

Let $\alpha, \frac{1}{\alpha}$ be the zeroes of $p(x)$, then

$$\alpha \cdot \frac{1}{\alpha} = \frac{4k}{k^2+4}$$

$$1 = \frac{4k}{k^2+4}$$

$$k^2 - 4k + 4 = 0$$

$$(k-2)^2 = 0$$

$$k = 2$$

Assertion is true Since, Reason is not correct for Assertion.

6. Assertion : $P(x) = 14x^3 - 2x^2 + 8x^4 + 7x - 8$ is a polynomial of degree 3.

Reason : The highest power of x in the polynomial $p(x)$ is the degree of the polynomial.

Ans : (d) Assertion (A) is false but reason (R) is true.

The highest power of x in the polynomial $p(x) = 14x^3 - 2x^2 + 8x^4 + 7x - 8$ is 4.

Degree of $p(x)$ is 4. So, A is incorrect but R is correct.

7. Assertion : $x^3 + x$ has only one real zero.

Reason : A polynomial of nth degree must have n real zeroes.

Ans : (c) Assertion (A) is true but reason (R) is false.

Reason is false [a polynomial of nth degree has at most x zeroes.]

Again, $x^3 + x = x(x^2+1)$

which has only one real zero

$(x = 0)$ $[x^2 + 1 \neq 0 \text{ for all } x \varepsilon R]$

Assertion is true.

8. Assertion : The sum and product of the zeros of a quadratic polynomial are $-\frac{1}{4}$ and $\frac{1}{4}$ respectively.

Then the quadratic polynomial is $4x^2 + x + 1$.

Reason : The quadratic polynomial whose sum and product of zeros are given is x^2-(sum of zeros). x + product of zeros.

Ans : (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

$$\text{Sum of zeros} = -\frac{1}{4} \text{ and}$$

$$\text{product of zeros} = \frac{1}{4}$$

Quadratic polynomial be $x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}$

$$\Rightarrow x^2 + \frac{1}{4}x + \frac{1}{4} \Rightarrow \frac{1}{4}(4x^2 + x + 1)$$

Quadratic polynomial be $4x^2 + x + 1$. So, both A and R are correct and R explains A.

9. Assertion : If both zeros of the quadratic polynomial $x^2 - 2kx + 2$ are equal in magnitude but opposite in sign then value of k is $\frac{1}{2}$.

Reason : Sum of zeros of a quadratic polynomial $ax^2 + bx + c$ is $\frac{-b}{a}$

Ans : (d) Assertion (A) is false but reason (R) is true.

As the polynomial is $x^2 - 2kx + 2$ and its zeros are equal but opposition sign

$$\text{sum of zeros} = 0 = \frac{-(-2k)}{1} = 0$$

$$\Rightarrow \quad 2k = 0 \Rightarrow k = 0$$

So, A is incorrect but R is correct.

10. Assertion : Degree of a zero polynomial is not defined.

Reason : Degree of a non-zero constant polynomial is '0'

Ans : (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

11. Assertion : The graph $y = f(x)$ is shown in figure, for the polynomial $f(x)$. The number of zeros of $f(x)$ is 4.

Reason : The number of zero of the polynomial $f(x)$. is the number of point of which $f(x)$ cuts or touches the axes.

Ans : (c) Assertion (A) is true but reason (R) is false.

As the number zero of polynomial $f(x)$ is the number of points at which $f(x)$ cuts (intersects) the x-axis and number of zero in the given figure is 4. So A is correct but R is incorrect.